Sesión Especial Número 3

Análisis Complejo y Teoría de Operadores

Organizadores

- Enrique Jordá Mora (Universidad Politécnica de Valencia)
- Fernando Pérez-González (Universidad de La Laguna)

Descripción

En esta sesión se expondrán trabajos de Análisis Complejo y Teoría de Operadores, dos de las ramas más clásicas del análisis matemático y están estrechamente relacionadas. De esta interrelación han surgido importantes resultados. Nuestro objetivo en esta sesión es explorar avances recientes en la interacción entre todas estas ramas, centrando nuestra atención principalmente en los siguientes temas: espacios de funciones analíticas y diferenciables, funciones armónicas, teoría geométrica de funciones, dinámica real y compleja, superficies de Riemann, teoría de pesos, etc..

Programa

Jueves, 7 de febrero (mañana)

11:30 – 12:00	José Bonet (Universidad Politécnica de Valencia) *Solid hulls and cores of weighted H^∞-spaces*
12:00 – 12:30	Guillermo Curbera (Universidad de Sevilla) *The multiplier algebra of the Cesàro space of Dirichlet series*
12:30 – 13:00	María José Beltrán (Universidad Jaume I) *Dynamics of weighted composition operators on spaces of continuous function*
13:00 – 13:30	María José González (Universidad de Cádiz) *Carleson measures on simply connected domains*

Jueves, 7 de febrero (tarde)

15:30 – 16:00	Luis Rodríguez Piazza (Universidad de Sevilla) *Convergence abscissa for Hardy-Orlicz spaces of Dirichlet series*
16:00 – 16:30	María José Martín (Universidad Autónoma de Madrid) *On the harmonic Möbius transformations*
16:30 – 17:00	Noel Merchán (Universidad de Málaga) *Hankel matrices acting on Dirichlet-type spaces*

17:30 – 18:00	Carme Cascante (Universitat de Barcelona) *Bilinear forms on potential spaces in the unit circle*
18:00 – 18:30	María del Carmen Reguera (University of Birmigham) *Bilinear embedding theorems in matrix weighted setting*

VIERNES, 8 de febrero (mañana)

10:00 – 10:30	Marina Murillo-Arcila (Universidad Jaume I) *Well-Posedness for degenerate third order equations with delay*
10:30 – 11:00	María Pilar Velasco (Universidad Politécnica de Madrid) *Fractional differential-difference equations: a discrete model for non-local and memory effects*
11:30 – 12:00	Dmitry V. Yakubovich (Universidad Autónoma de Madrid) *Perturbations of normal operators and questions of geometric measure theory*
12:00 – 12:30	José G. Llorente *Picard's theorem and the range of harmonic maps*

Dynamics of weighted composition operators on spaces of continuous functions

María José Beltrán Meneu

Universitat Jaume I/ Jaume I University

mmeneu@uji.es

Abstract. Our study is focused on dynamics of weighted composition operators $C_{w,\varphi}$ defined on a locally convex space $E \hookrightarrow (C(X), \tau_p)$ with X being a topological Hausdorff space containing at least two different points and such that the evaluations $\{\delta_x : \ x \in X\}$ are linearly independent in E'. We prove, when X is compact and E is a Banach space containing a nowhere vanishing function, that $C_{w,\varphi}$ is never weakly supercyclic on E. We also provide sufficient conditions for the symbol φ that ensure no τ_p-supercyclicity. For the case $X = \overline{\mathbb{D}}$ we prove that an isometric weighted composition operator can never be τ_p-supercyclic. If the symbol φ lies in the unit ball of the disk algebra $A(\mathbb{D})$, we show that $C_{w,\varphi}$ can never be τ_p-supercyclic neither on $C(\mathbb{D})$ nor on $A(\mathbb{D})$. For the case $X = \partial\mathbb{D}$ we obtain conditions which ensure $C_{w,\varphi}$ is not τ_p-supercyclic. Finally, from our work on weighted composition operators we obtain Ansari-Bourdon type results and conditions on the spectrum for arbitrary weakly supercyclic operators.

Joint work with Enrique Jordá y Marina Murillo-Arcila
Partially supported by MEC, MTM2016-76647-P and MTM2016-75963-P

Solid hulls and cores of weighted H^∞-spaces

José Bonet

Instituto Universitario de Matemática Pura y Aplicada IUMPA,
Universitat Politècnica de València,

jbonet@mat.upv.es

Abstract. We determine the solid hull and solid core of weighted Banach spaces H_v^∞ of analytic functions functions f such that $v|f|$ is bounded, both in the case of the holomorphic functions on the disc and on the whole complex plane, for a very general class of strictly positive, continuous, radial weights v. Precise results are presented for concrete weights on the disc that could not be treated before. It is also shown that if H_v^∞ is solid, then the monomials are an (unconditional) basis of the closure of the polynomials in H_v^∞. As a consequence H_v^∞ does not coincide with its solid hull and core in the case of the disc. An example shows that this does not hold for weighted spaces of entire functions.

Joint work with W. Lusky (Paderborn, Germany) and J. Taskinen (Helsinki, Finland).

Bilinear forms on potential spaces in the unit circle

Carme Cascante

Universitat de Barcelona

cascante@ub.edu

Abstract. We characterize the boundedness on the product of Sobolev spaces $H^s(\mathbb{T}) \times H^s(\mathbb{T})$ on the unit circle, $0 < s < 1/2$, of the bilinear form Λ_b, with symbol $b \in H^s(\mathbb{T})$, given by

$$\Lambda_b(\varphi, \psi) = \int_{\mathbb{T}} ((-\Delta)^s + I)(\varphi\psi)(\eta) b(\eta) d\sigma(\eta).$$

The space $H^s(\mathbb{T})$ coincides with the completion of $\mathcal{C}^\infty(\mathbb{T})$ with respect to the norm $\|\varphi\|_{L^2(\mathbb{T})} + \|(-\Delta)^s \varphi\|_{L^2(\mathbb{T})}$, where $(-\Delta)^s$ is the fractional laplacian defined, up to a constant, by

$$(-\Delta)^s(\varphi)(\zeta) = P.V. \int_{\mathbb{T}} \frac{\varphi(\zeta) - \varphi(\eta)}{|\zeta - \eta|^{1+2s}} d\sigma(\eta).$$

In turn, this space coincides with the space of Riesz potentials $I_s(L^2(\mathbb{T}))$, where I_s is the Riesz kernel defined, up to a constant, by $I_s(\zeta, \eta) = \frac{1}{|1-\zeta\overline{\eta}|^{1-s}}$.
This bilinear problem is equivalent to the characterization of the functions $c \in L^2(\mathbb{T})$ that are trace measures (that may change sign) for the space $H^s(\mathbb{T})$, i.e.,

$$\left| \int_{\mathbb{T}} |\varphi|^2 c \, d\sigma \right| \lesssim \|\varphi\|^2_{H^s(\mathbb{T})},$$

We avoid some of the difficulties when dealing with fractional laplacians by considering an equivalent bilinear problem on a subspace of a weighted Sobolev space on the unit disc, consisting of extensions of functions on $H^s(\mathbb{T})$ by a generalized Poisson operator.

Joint work with Joaquim M. Ortega

The multiplier algebra of the Cesàro space of Dirichlet series

Guillermo P. Curbera

Universidad de Sevilla

curbera@us.es

Abstract. We consider the space $\mathcal{H}(ces_p)$, for $1 < p < \infty$, consisting of all Dirichlet series

$$\sum_{n=1}^{\infty} a_n n^{-s}$$

whose coefficients belong to the Cesàro sequence space ces_p, consisting of all complex sequences $a = (a_n)_{n=1}^{\infty}$ whose absolute Cesàro means are in ℓ^p, that is,

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} \sum_{k=1}^{n} |a_k| \right)^p < \infty.$$

Contrary to what occurs with other known non-trivial examples of spaces of Dirichlet series, the multiplier algebra of $\mathcal{H}(ces_p)$ is not a space of bounded Dirichlet series, but the weighted ℓ^1 space of Dirichlet series

$$\mathcal{A}^{1/q} := \left\{ \sum_{n=1}^{\infty} b_n n^{-s} : \sum_{n=1}^{\infty} |b_n| n^{-1/q} < \infty \right\},$$

where $1/p + 1/q = 1$.

Joint work with Jorge Bueno-Contreras and Olvido Delgado.

Carleson measures on simply connected domains

María José González

Universidad de Cádiz

majose.gonzalez@uca.es

Abstract. Carleson characterized the positive measures μ on the unit disc $\mathbb{D}$ for which the Hardy space $\mathbb{H}^p(\mathbb{D})$ embeds continuously in $L^p(\mu)$. This theorem has led to various generalizations, including similar results for the weighted Bergman spaces A^p_α, $\alpha > -1$. In this talk we will review these classical results and show how similar characterizations hold on more general domains of the plane.

Picard's theorem and the range of harmonic maps

José G. Llorente

Universitat Autònoma de Barcelona

jgllorente@mat.uab.cat

Abstract. Picard's little theorem is one of the most representative results in classical Complex Analysis. Each approach (Montel's theorem, Schottky or Bloch theorems, the modular function, curvature of metrics, brownian motion...) is highly non-trivial and has contributed in a significant way to the development of Geometric Function Theory. In 1994 J. Lewis, motivated by versions of the theorem for more general classes of functions, obtained yet a conceptually simple, purely "harmonic"proof of Picard's theorem.

The starting point in Lewis' proof is the observation that if f is analytic, entire and omits the values 0 and 1 then $u = \log|f|$ and $v = \log|f-1|$ are harmonic, entire and its values satisfy certain relations which force them to be constant. In the talk we will review Lewis' argument and present some related conjectures and positive (or negative) results.

On the harmonic Möbius transformations

María J. Martín

Universidad Autónoma de Madrid

mariaj.martin@uam.es

Abstract. It is well-known that two locally univalent analytic functions have equal Schwarzian derivative if and only if each one of them is a pre-composition of the other with a non-constant Möbius transformation.
The main goal in this talk is to identify completely the relationship between two locally univalent *harmonic* mappings with equal (harmonic) Schwarzian derivative. The answer is expressed as a dichotomy between two different situations. The absence of governing second order differential equations, which are extremely helpful in the analytic case, makes this task a lot more difficult. The computations needed to deduce the main result are, thus, much more involved that in the analytic case and rely heavily on the invariance properties of the harmonic Schwarzian derivative.

Joint work with Rodrigo Hernández.

Hankel matrices acting on Dirichlet-type spaces

Noel Merchán

Universidad de Málaga

noel@uma.es

Abstract. If μ is a positive Borel measure on the interval $[0,1)$ we let $\mathcal{H}_\mu$ be the Hankel matrix $\mathcal{H}_\mu = (\mu_{n,k})_{n,k\geq 0}$ with entries $\mu_{n,k} = \mu_{n+k}$, where, for $n = 0,1,2,\dots$, μ_n denotes the moment of orden n of μ. This matrix induces formally the operator

$$\mathcal{H}_\mu(f)(z) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{\infty}\mu_{n,k}a_k\right)z^n$$

on the space of all analytic functions $f(z) = \sum_{k=0}^{\infty} a_k z^k$, in the unit disc $\mathbb{D}$. This is a natural generalization of the classical Hilbert operator.
In this work we study the action of the operators $\mathcal{H}_\mu$ to the Dirichlet-type spaces

$$\mathcal{D}_\alpha^p = \left\{ f \in \mathcal{H}ol(\mathbb{D}) \,:\, \int_{\mathbb{D}} (1-|z|^2)^\alpha |f'(z)|^p \, dA(z) \, < \, \infty \right\}.$$

Referencias

[1] D. Girela and N. Merchán, *Hankel matrices acting on the Hardy space H^1 and on Dirichlet spaces*, preprint (available at https://arxiv.org/pdf/1804.02227.pdf)

Joint work with Daniel Girela

Well-Posedness for degenerate third order equations with delay

Marina Murillo Arcila

Universitat Jaume I / University Jaume I

murillom@uji.es

Abstract. In this talk, we study well-posedness for the following third-order in time equation with delay

$$\alpha(Mu')''(t) + (Nu')'(t) = \beta Au(t) + \gamma Bu'(t) + Gu'_t + Fu_t + f(t), \quad t \in [0, 2\pi]$$

where α, β, γ are real numbers, A and B are linear operators defined on a Banach space X with domains $D(A)$ and $D(B)$ such that $D(A) \cap D(B) \subset D(M) \cap D(N)$; $u(t)$ is the state function taking values in X and $u_t : (-\infty, 0] \to X$ defined as $u_t(\theta) = u(t+\theta)$ for $\theta < 0$ belongs to an appropriate phase space where F and G are bounded linear operators. Using operator-valued Fourier multipliers techniques we provide optimal conditions for well-posedness of our equation in periodic Lebesgue-Bochner spaces $L^p(\mathbb{T}, X)$, periodic Besov spaces $B^s_{p,q}(\mathbb{T}, X)$ and periodic Triebel-Lizorkin spaces $F^s_{p,q}(\mathbb{T}, X)$. A novel application to an inverse problem is given.

Referencias

[1] Conejero, A., Lizama, C., Murillo-Arcila, M and Seoane-Sepúlveda,J.B., Well-Posedness for degenerate third order equations with delay and applications to inverse problems, to appear in Israel Journal of Mathematics.

Joint work with J.Alberto Conejero, Carlos Lizama and J.B. Seoane-Sepúlveda.

Bilinear embedding theorems in matrix weighted setting

María del Carmen Reguera

University of Birmingham

m.reguera@bham.ac.uk

Abstract. We prove a bilinear embedding theorem in the matrix weighted setting with scalar coefficients. This is a step forward to understanding bilinear embedding estimates in the matrix weighted setting. The general case is still an open question, one that will provide very sought after sharp weighted estimates in this setting.

Joint work with with S. Pott and S. Petermichl

Convergence abscissa for Hardy-Orlicz spaces of Dirichlet series

Luis Rodríguez Piazza

Universidad de Sevilla

piazza@us.es

Abstract. The Hardy spaces of Dirichlet series $\mathcal{H}^p$ ($1 \leq p \leq \infty$) have been introduced by Hedenmalm, Lindqvist and Seip [1] when $p = 2$ and $p = \infty$, and by Bayart [2] for the general values of p. It was proved that, for p finite, every Dirichlet series in $\mathcal{H}^p$ converges in the half-plane

$$\mathbb{C}_{1/2} = \{z \in \mathbb{C} : \Re z > 1/2\},$$

and that $1/2$ can not be improved to a smaller number (there is no $\theta < 1/2$ such that every Dirichlet series in $\mathcal{H}^p$ converges in $\mathbb{C}_\theta$). Hence we can say that, for p finite, the convergence abscissa of $\mathcal{H}^p$ is $1/2$. On the other hand it is known that the convergence abscissa of $\mathcal{H}^\infty$ is 0.

In our talk we will introduce the Orlicz version of Hardy spaces of Dirichlet series $\mathcal{H}^\psi$. We will focus on the case $\psi = \psi_q(t) = \exp(t^q) - 1$ and will compute the convergence abscissa for these spaces, obtaining that this abscissa is $\min\{1/2, 1/q\}$. This fills the gap between the abscissa of $\mathcal{H}^p$, for p finite, and the one of $\mathcal{H}^\infty$, answering a question raised by Hedenmalm [3].

Referencias

[1] H. Hedenmalm, P. Lindqvist and K. Seip, *A Hilbert space of Dirichlet series and systems of dilated functions in $L^2(0,1)$*, Duke Math. J. **86** (1997) 1–37.

[2] F. Bayart *Hardy spaces of Dirichlet series and their composition operators*, Monatshefte für Mathematik, **136** (2002), 203–236.

[3] H. Hedenmalm, *Dirichlet series and functional analysis*, The legacy of Niels Henrik Abel, 673–684, Springer, Berlin, 2004.

Fractional differential-difference equations: a discrete model for non-local and memory effects

M. Pilar Velasco

Universidad Politécnica de Madrid

mp.velasco@upm.es

Abstract. The study of existence and qualitative properties of time-discrete solutions for fractional equations is a matter of great deal of interest in the last decade. In spite of the significant increase of research in this area, there are still many open questions regarding fractional difference equations. In particular, the study of well posedness for the semidiscretisation in time of evolution equations, involving bounded linear operators defined on Banach spaces remains largely untreated.

These models are connected with numerical methods for partial differential equations, integro-differential equations, evolution equations with memory and lattices models. The discrete fractional equations is also a promising tool for several biological and physical applications where the memory effect appears.

We develop an operator-theoretical method for the analysis on well posedness of differential-difference equations that can be modeled in the form

$$\left\{ \begin{array}{rl} \Delta^\alpha u(n) & = Au(n) + f(n, u(n)), \quad n \in \mathbb{N}_0, \ 0 < \alpha \leq 1; \\ u(0) & = u_0, \end{array} \right.$$

where Δ^α is a fractional difference operator and A is a linear is a closed linear operator defined on a Banach space X.

Referencias

[1] L. Abadias, C. Lizama, P.J. Miana, M.P. Velasco. Cesaro sums and algebra homomorphisms of bounded operators. Israel J. Math. 216 (1) (2016), 471-505.

[2] C. Lizama, M.P. Velasco. Weighted bounded solutions for a class of nonlinear fractional equations. Fract. Calc. Appl. Anal. 19 (4) (2016), 1010-1030.

[3] L. Abadias, C. Lizama, P.J. Miana, M.P. Velasco. On well-posedness of vector-valued fractional differential-difference equations. Discrete and Continuous Dynamical Systems. In press.

Joint work with Carlos Lizama (Universidad de Santiago de Chile)
Partially supported by the Project ESP2016-79135-R (MCYTS)

Perturbations of normal operators and questions of geometric measure theory

Dmitry V. Yakubovich

Universidad Autónoma de Madrid

dmitry.yakubovich@uam.es

Abstract. Let N be a bounded normal operator on a separable Hilbert space and let μ stand for its scalar spectral measure. The spectral nature of the perturbation $T = N + K$, where K is a sufficiently "smooth" compact operator, will be discussed. We are interested in the existence of invariant subspaces, decomposability and other questions.
We introduce the perturbation operator-valued function of T, defined in the whole complex plane, except for a certain thin set, and explain its role. We discuss the dependence of the answers on geometric properties of μ.
The case when μ is absolutely continuous with respect to the area measure has been considered in [1] in 1993; this is the case of μ of "dimension" 2. A quotient model for T, constructed in terms of certain vector-valued Sobolev classes of functions, was established in this work. The case of a discrete measure μ (that is, of a diagonalizable operator N) has been studied more recently in a series of papers by C. Foias and his coauthors (see [2]). This can be seen as the case of zero dimension. The case when μ is of "dimension" 1 (that is, when μ behaves like the arc length measure) seems to be the most difficult one, whereas many cases of fractional dimension are easier

Referencias

[1] D. Yakubovich, Spectral properties of smooth perturbations of normal operators with planar Lebesgue spectrum, Indiana Univ. Math. J. 42, no. 1 (1993), 55–83.

[2] C. Foias, I. Jung, E. Ko, C. Pearcy, Spectral decomposability of rank-one perturbations of normal operators, J. Math. Anal. Appl. 375 (2011), 602–609.

This is a joint work in progress with Mihai Putinar